4.4 Prove Triangles Congruent by SAS and HL

Before	You used the SSS Congruence Postulate.
Now	You will use sides and angles to prove congruence.
Why?	So you can show triangles are congruent, as in Ex. 33.

Key Vocabulary
- **leg of a right triangle**
- **hypotenuse**

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

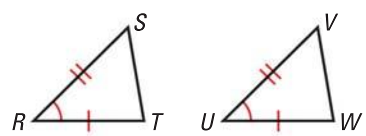
Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

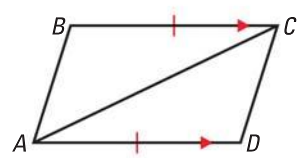


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

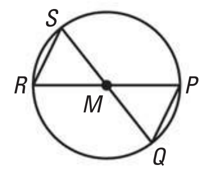


	STATEMENTS	REASONS
S	1. $\overline{BC} \cong \overline{DA}$	1. Given
	2. $\overline{BC} \parallel \overline{AD}$	2. Given
A	3. $\angle BCA \cong \angle DAC$	3. Alternate Interior Angles Theorem
S	4. $\overline{AC} \cong \overline{CA}$	4. Reflexive Property of Congruence
	5. $\triangle ABC \cong \triangle CDA$	5. SAS Congruence Postulate

WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



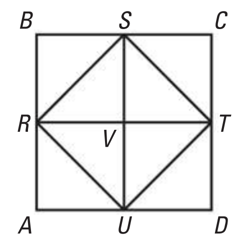
Solution

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so MP, MQ, MR, and MS are all equal.

▶ $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Postulate.

✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $ABCD$ is a square with four congruent sides and four right angles. R, S, T, and U are the midpoints of the sides of $ABCD$. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.



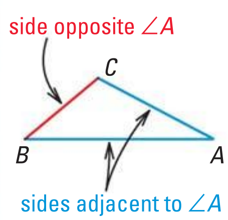
1. Prove that $\triangle SVR \cong \triangle UVR$.
2. Prove that $\triangle BSR \cong \triangle DUT$.

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

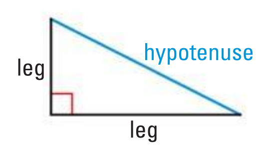
Therefore, SSA is *not* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

READ VOCABULARY

The two sides of a triangle that form an angle are *adjacent* to the angle. The side not adjacent to the angle is *opposite* the angle.



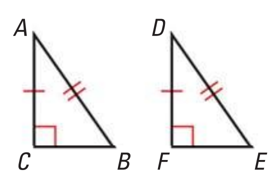
RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

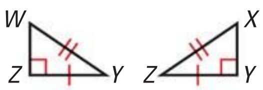


$\triangle ABC \cong \triangle DEF$

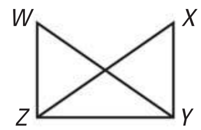
Proofs: Ex. 37, p. 439; p. 932

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

USE DIAGRAMS
If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



Write a proof.

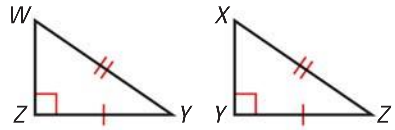


GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$

Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS
H 1. $\overline{WY} \cong \overline{XZ}$	1. Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	2. Given
3. $\angle Z$ and $\angle Y$ are right angles.	3. Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	4. Definition of a right triangle
L 5. $\overline{ZY} \cong \overline{YZ}$	5. Reflexive Property of Congruence
6. $\triangle WYZ \cong \triangle XZY$	6. HL Congruence Theorem

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EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?

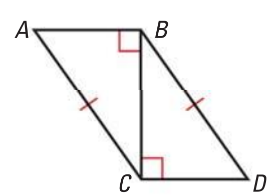
Solution

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two sides and their included angle are congruent.

▶ You can use the SAS Congruence Postulate to conclude that $\triangle PQR \cong \triangle PSR$.

✓ GUIDED PRACTICE for Examples 3 and 4

Use the diagram at the right.



3. Redraw $\triangle ACB$ and $\triangle DBC$ side by side with corresponding parts in the same position.
4. Use the information in the diagram to prove that $\triangle ACB \cong \triangle DBC$.

4.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 23, and 39

SKILL PRACTICE

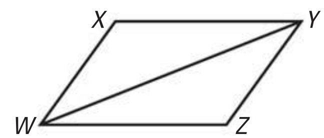
1. **VOCABULARY** Copy and complete: The angle between two sides of a triangle is called the _?_ angle.

2. ★ **WRITING** *Explain* the difference between proving triangles congruent using the SAS and SSS Congruence Postulates.

EXAMPLE 1 on p. 240 for Exs. 3–15

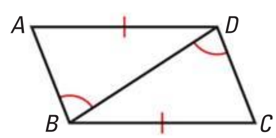
NAMING INCLUDED ANGLES Use the diagram to name the included angle between the given pair of sides.

3. $\overline{XY}$ and $\overline{YW}$
4. $\overline{WZ}$ and $\overline{ZY}$
5. $\overline{ZW}$ and $\overline{YW}$
6. $\overline{WX}$ and $\overline{YX}$
7. $\overline{XY}$ and $\overline{YZ}$
8. $\overline{WX}$ and $\overline{WZ}$

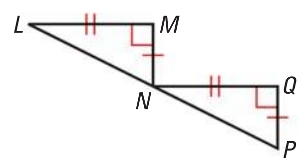


REASONING Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

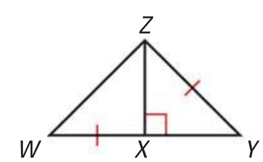
9. $\triangle ABD$, $\triangle CDB$



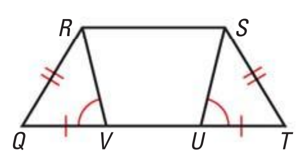
10. $\triangle LMN$, $\triangle NQP$



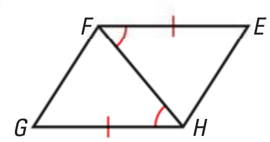
11. $\triangle YXZ$, $\triangle WXZ$



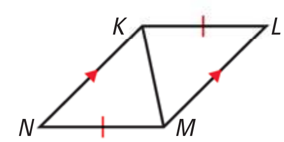
12. $\triangle QRV$, $\triangle TSU$



13. $\triangle EFH$, $\triangle GHF$



14. $\triangle KLM$, $\triangle MNK$



15. ★ **MULTIPLE CHOICE** Which of the following sets of information does not allow you to conclude that $\triangle ABC \cong \triangle DEF$?

   Ⓐ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$

   Ⓑ $\overline{AB} \cong \overline{DF}$, $\overline{AC} \cong \overline{DE}$, $\angle C \cong \angle E$

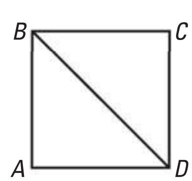
   Ⓒ $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, $\overline{BA} \cong \overline{DE}$

   Ⓓ $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$

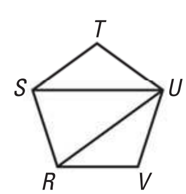
EXAMPLE 2 on p. 241 for Exs. 16–18

APPLYING SAS In Exercises 16–18, use the given information to name two triangles that are congruent. *Explain* your reasoning.

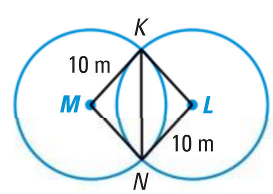
16. *ABCD* is a square with four congruent sides and four congruent angles.



17. *RSTUV* is a regular pentagon.

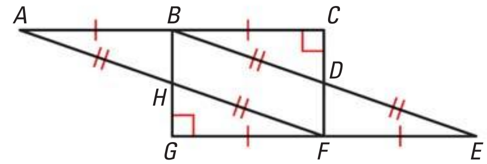


18. $\overline{MK} \perp \overline{MN}$ and $\overline{KL} \perp \overline{NL}$.



EXAMPLE 3 on p. 242 for Ex. 19

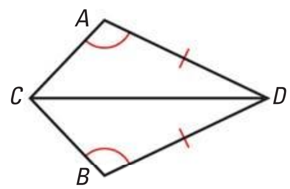
19. **OVERLAPPING TRIANGLES** Redraw $\triangle ACF$ and $\triangle EGB$ so they are side by side with corresponding parts in the same position. *Explain* how you know that $\triangle ACF \cong \triangle EGB$.



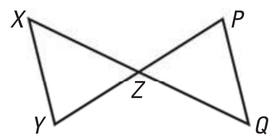
EXAMPLE 4 on p. 242 for Exs. 20–22

REASONING **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.**

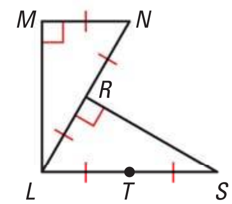
20.



21. Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$.

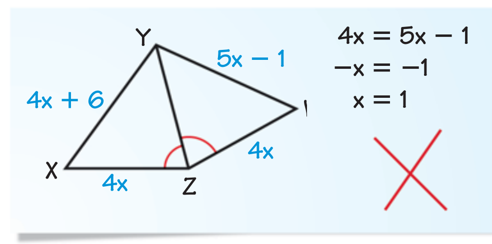


22.



23. ★ **WRITING** Suppose both pairs of corresponding legs of two right triangles are congruent. Are the triangles congruent? *Explain*.

24. **ERROR ANALYSIS** *Describe* and correct the error in finding the value of x.

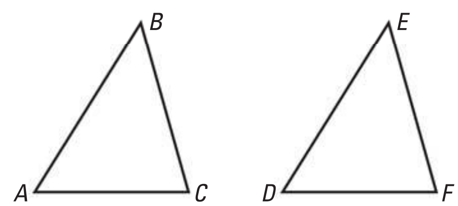


USING DIAGRAMS **In Exercises 25–27, state the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate.**

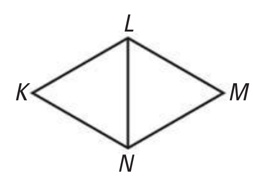
25. **GIVEN** ▶ $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, _?_ ≅ _?_
Use the SSS Congruence Postulate.

26. **GIVEN** ▶ $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, _?_ ≅ _?_
Use the SAS Congruence Postulate.

27. **GIVEN** ▶ $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, _?_ ≅ _?_
Use the SAS Congruence Postulate.

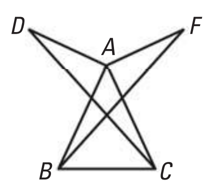


28. **USING ISOSCELES TRIANGLES** Suppose $\triangle KLN$ and $\triangle MLN$ are isosceles triangles with bases $\overline{KN}$ and $\overline{MN}$ respectively, and $\overline{NL}$ bisects $\angle KLM$. Is there enough information to prove that $\triangle KLN \cong \triangle MLN$? *Explain*.



29. **REASONING** Suppose M is the midpoint of $\overline{PQ}$ in $\triangle PQR$. If $\overline{RM} \perp \overline{PQ}$, *explain* why $\triangle RMP \cong \triangle RMQ$.

30. **CHALLENGE** Suppose $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AF}$, $\overline{AD} \perp \overline{AB}$, and $\overline{AF} \perp \overline{AC}$. *Explain* why you can conclude that $\triangle ACD \cong \triangle ABF$.



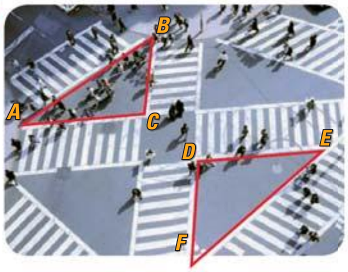
○ = WORKED-OUT SOLUTIONS on p. WS1

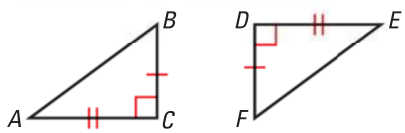
★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

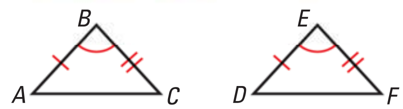
CONGRUENT TRIANGLES **In Exercises 31 and 32, identify the theorem or postulate you would use to prove the triangles congruent.**

31.





32.



33. SAILBOATS Suppose you have two sailboats. What information do you need to know to prove that the triangular sails are congruent using SAS? using HL?

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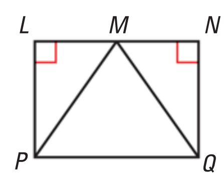
EXAMPLE 3 on p. 242 for Ex. 34

34. DEVELOPING PROOF Copy and complete the proof.



GIVEN ▶ Point M is the midpoint of $\overline{LN}$.
$\triangle PMQ$ is an isosceles triangle with base $\overline{PQ}$.
$\angle L$ and $\angle N$ are right angles.

PROVE ▶ $\triangle LMP \cong \triangle NMQ$

STATEMENTS	REASONS
1. $\angle L$ and $\angle N$ are right angles.	1. Given
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.	2. ___?___
3. Point M is the midpoint of $\overline{LN}$.	3. ___?___
4. ___?___	4. Definition of midpoint
5. $\triangle PMQ$ is an isosceles triangle.	5. Given
6. ___?___	6. Definition of isosceles triangle
7. $\triangle LMP \cong \triangle NMQ$	7. ___?___

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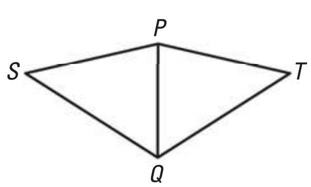
PROOF **In Exercises 35 and 36, write a proof.**

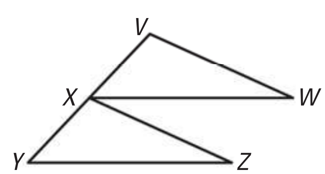
35. GIVEN ▶ $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

PROVE ▶ $\triangle SPQ \cong \triangle TPQ$



36. GIVEN ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$

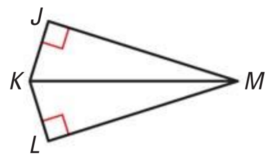
PROVE ▶ $\triangle VXW \cong \triangle XYZ$



PROOF **In Exercises 37 and 38, write a proof.**

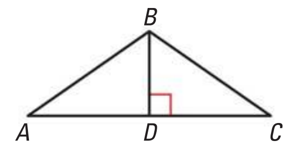
37. **GIVEN** ▶ $\overline{JM} \cong \overline{LM}$

PROVE ▶ $\triangle JKM \cong \triangle LKM$



38. **GIVEN** ▶ D is the midpoint of $\overline{AC}$.

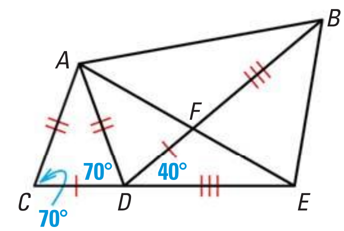
PROVE ▶ $\triangle ABD \cong \triangle CBD$



39. ★ **MULTIPLE CHOICE** Which triangle congruence can you prove, then use to prove that $\angle FED \cong \angle ABF$?

Ⓐ $\triangle ABE \cong \triangle ABF$ Ⓒ $\triangle AED \cong \triangle ABD$

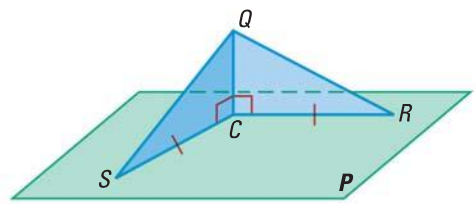
Ⓑ $\triangle ACD \cong \triangle ADF$ Ⓓ $\triangle AEC \cong \triangle ABD$



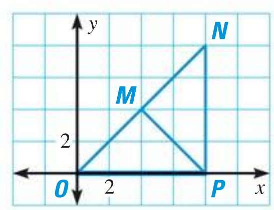
40. **PROOF** Write a two-column proof.

GIVEN ▶ $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$

PROVE ▶ $\triangle QCR \cong \triangle QCS$



41. **CHALLENGE** *Describe* how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then show that the triangles are congruent using the SAS Congruence Postulate without measuring any angles. *Compare* the two methods.



MIXED REVIEW

Draw a figure that fits the description. *(p. 42)*

42. A pentagon that is not regular.

43. A quadrilateral that is equilateral but not equiangular.

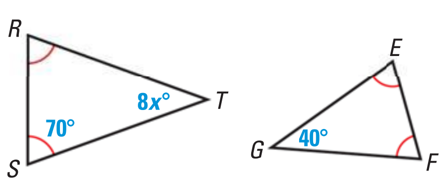
Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. *(p. 180)*

44. $P(3, -1), y = -x + 2$ **45.** $P(3, 3), y = \frac{1}{3}x + 2$ **46.** $P(-4, -7), y = -5$

PREVIEW
Prepare for Lesson 4.5 in Exs. 47–48.

Find the value of x. *(p. 225)*

47.



48.